Risk Prioritization: The Analytic Hierarchy Process Approach

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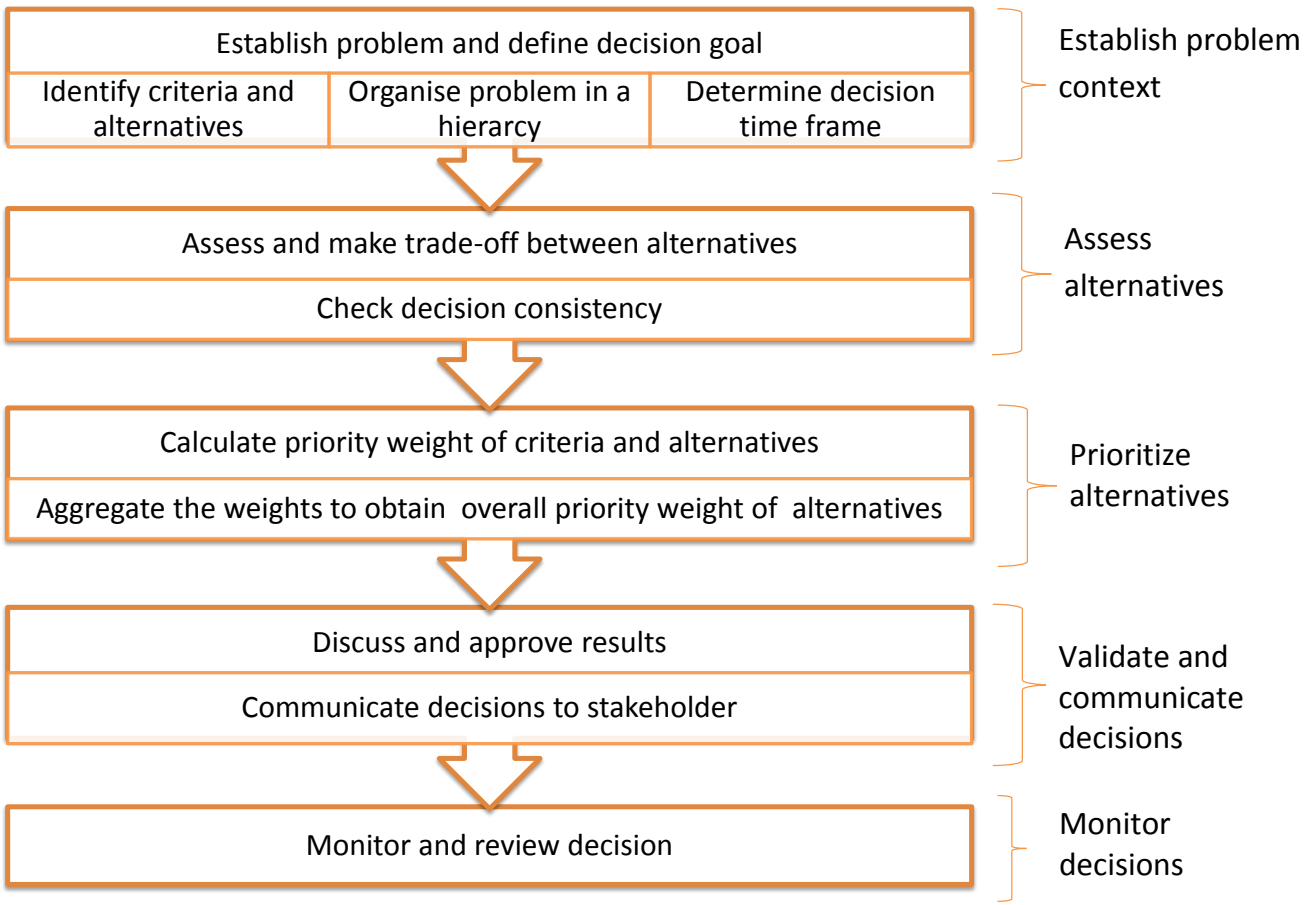
Presentation goals

- Introduction to Risk Prioritization (RP).
- RP decision making steps
- Demonstrate how RP rank risks.
- Appendix
  - Decision Consistency
  - Log Least Square Method to calculate priority vector.

What is RP?

- RP is a decision making tool to prioritize risks or prioritize alternatives under risky or uncertain situations.
- RP combines decision making theory and risk analysis in making judgement, and use mathematics to quantify soft decisions.
- RP exploits Analytic Hierarchy Process (AHP).
- AHP was developed by Thomas L. Saaty in 1970's.
- AHP is a multi criteria decision making method for *structuring*, *measuring* and *synthesizing* a complex problem.

RP Decision Making Steps



Step 1: Establish Problem Context

1.1 Establish problem and define decision goal. Discuss and define the problem to establish and understand the problem and determine decision goal.

1.2 Identify criteria and alternatives. Alternatives can be either risks, risk actions or policies to be prioritize. Alternatives can also be options under uncertain situations. Criteria are factors or attributes used to assess alternatives.

1.3 Organize the information in a hierarchy. The top level is the decision goal. Followed by criteria and alternatives.

1.4 Determine decision time frame. Decide the duration of the risk assessment.

Step 2: Assess Alternatives

2.1 Assess and make trade-off between alternatives. RP use pairwise comparison to make trade-off and asses the importance of alternatives.

2.2 Check decision consistency. RP use subjective judgement to prioritize risks . One of the difficulties of subjective judgement is maintaining consistent decision. RP use consistency ratio (CR) to measure consistency.

Step 3: Prioritize Alternatives

3.1 Calculate priority weight of criteria & alternatives. RP uses Log Least Square Method (LLSM) to obtain priority vector of criteria and alternatives.

3.2 Aggregate the weights - overall priority weight of alternatives. The priority vector of each criterion are combined into a single matrix. The new matrix is multiplied by the priority vector of the criteria and normalized to obtain the global priority vector

Step 4: Validate and Communicate Decision

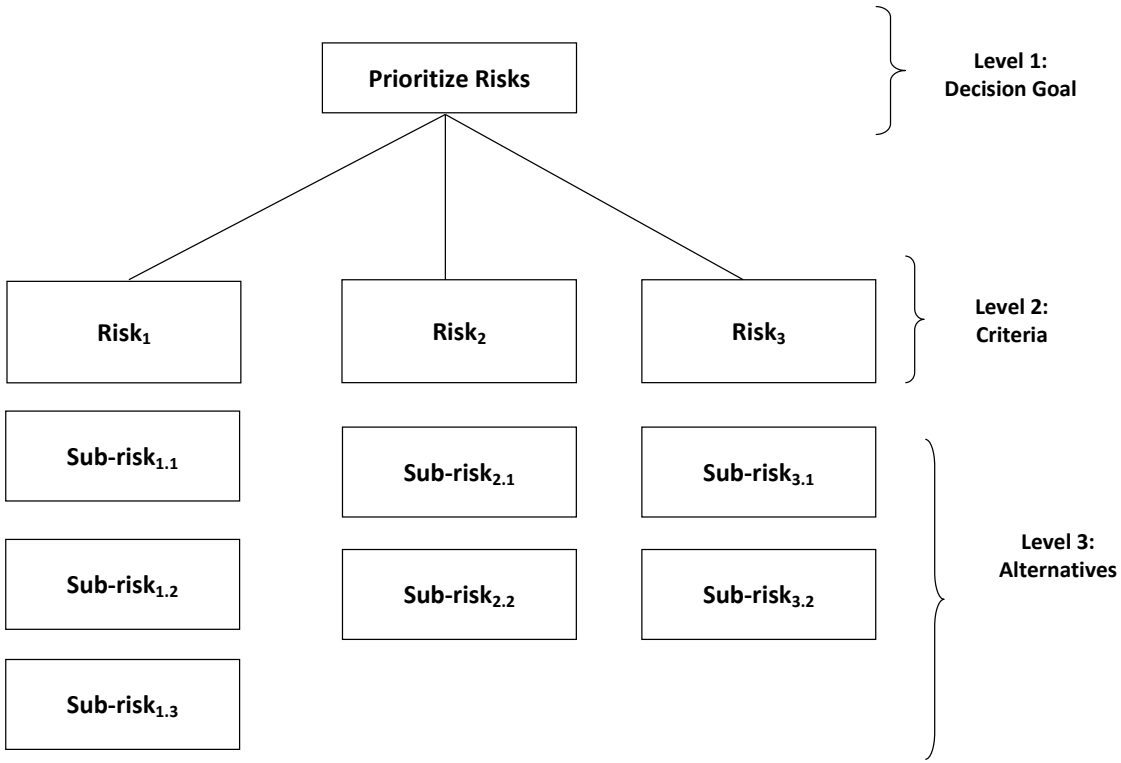
4.1 Discuss and approved results. Discuss results with board, management team or stakeholders.

4.2 Communicate decision to stakeholders.

Step 5: Monitor Decision

5.1 Monitor and review decisions. Risk evolve. Factors relevant to a problem might change. Monitor problem environment and review decision if necessary.

Building a Hierarchy



Pairwise Comparison Scale

Value	Definition
1	Equal important or preferred
3	Moderately important or preferred
5	Strongly important or preferred
7	Very strong important or preferred
9	Extremely important or preferred of one over another
Reciprocals	Reciprocals for inverse comparison

Pairwise Comparison Judgement

Paired comparison of sub-risks

Risk 1	Sub-Risk 1.1	Sub-Risk 1.2	Sub-Risk 1.3
Sub-Risk 1.1	1	5	7
Sub-Risk 1.2	1/5	1	3
Sub-Risk 1.3	1/7	1/3	1

In terms of Risk 1:
Sub-Risk 1.1 is strongly more important compared to Sub-Risk 1.2.
Sub-Risk 1.1 is very strongly more important compared to Sub-Risk 1.3.
Sub-Risk 1.2 is moderately more important compared to Sub-Risk 1.3

Pairwise Comparison Judgement

Pairwise comparison of Risks

	Risk 1	Risk 2	Risk 3
Risk 1	1	3	9
Risk 2	1/3	1	5
Risk 3	1/9	1/5	1

Risk 1 is moderately important compared to Risk 2
Risk 1 is extremely important compared to Risk 3
Risk 2 is strongly important compared to Risk 3

Calculate Priority Vector (PV) of Sub-Risks

Risk 1	Sub-risk 1.1	Sub-risk 1.2	Sub-risk 1.3	PV
Sub-risk 1.1	1	5	7	0.73
Sub-risk 1.2	1/5	1	3	0.19
Sub-risk 1.3	1/7	1/3	1	0.08

In terms of Risk 1, Sub-risk 1.1 is most important..

Calculate Priority Vector (PV) of Risk

	Risk 1	Risk 2	Risk 3	PV
Risk 1	1	3	9	0.67
Risk 2	1/3	1	5	0.27
Risk 3	1/9	1/5	1	0.06

Risk 1 is the most important risk.

Synthesizing Priority Vector

To get overall ranking of sub-risk. Priority vector of sub-risk for each risk are multiplied by priority vector of risk to obtain the overall or global priority vector (GPV).

	Risk 1 0.67	Risk 2 0.27	Risk 3 0.06	GPV
Risk 1.1	0.73			0.49
Risk 1.2	0.19			0.13
Risk 1.3	0.08			0.05
Risk 2.1		0.55		0.15
Risk 2.2		0.45		0.12
Risk 3.1			0.7	0.04
Risk 3.2			0.3	0.02

RP Strengths

- Making trade-off.
- Structuring and decomposing complex problem.
- Broad framing
- Transparent and traceable decision.
- Measure decision consistency.
- Easy to use and understand.

RP Shortcomings

- How big is the hierarchy?
- Decision fatigue - too many pairwise comparisons.
- Model validation.

Appendix: Decision Consistency

The accuracy of the decisions is measured by computing Consistency Ratio (CR) and Consistency Index (CI).

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RCI}}\ , \quad \mathrm{CI} = \frac{\lambda_{\max} - n}{n-1}$$

RCI is a pre-defined average random index derived from a sample size of 500 of randomly generated reciprocal matrices. $\lambda_{\max}$ equals the maximum eigenvalue of the judgement matrix.

- CR equals or less that 10% is acceptable indicating the judgement is consistent.
- CR exceeding 10% indicates inconsistency in judgement. Risks are not properly compared.
- CR exceeding 10% requires revising the pairwise comparison judgement.

Appendix: The Log Least Square Method

The LLSM (Jong 1984; Crawford and Williams 1985; Crawford 1987) is to minimize the following equation:

$$min \sum_{j=1}^{n} \sum_{i=1}^{n} (log(a_{ij}) - log(\frac{w_i}{w_j}))^2$$

Writing $log(a_{ij}) = Y_{ij}$ and $p_i = log(w_i)$ produce the equation

$$min \sum_{j=1}^{n} \sum_{i=1}^{n} (Y_{ij} - p_i + p_j)^2$$

under the condition

$$\sum_{j=1}^{n} p_j = 0, \quad Y_{ij} = -Y_{ji} \quad , Y_{ii} = 0$$

Appendix: LLSM Continue

Taking partial derivatives set equal to zero:

$$-2\sum(Y_{ij} - p_i + p_j) = 0$$

$$-2(\sum_{i=1}^{n} Y_{ij} - np_i + \sum_{j=1}^{n} p_j) = 0$$

$$\sum_{i=1}^{n} Y_{ij} = np_i$$

$$p_i = \frac{\sum_{i=1}^{n} Y_{ij}}{n}$$

$$log(w_i) = \frac{\sum_{i=1}^{n} log(a_{ij})}{n}$$

Appendix: LLSM Continue

The antilog is the LLSM solution. The minimize vector is given by

$$w_i = \prod_{i=1}^{n} (a_{ij})^{1/n}$$

Consider the following pairwise comparison Matrix A:

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 9 \\ 1/3 & 1 & 5 \\ 1/9 & 1/5 & 1 \end{pmatrix}$$

Appendix: LLSM Continue

The calculation of the priority vector for **A** is as the following:

$$\mathbf{w_i} = \begin{pmatrix} (1 \times 3 \times 9)^{1/3} = 3 \\ (1/3 \times 1 \times 5)^{1/3} = 0.18 \\ (1/9 \times 1/5 \times 1)^{1/3} = 0.28 \end{pmatrix}$$

The normalized priority for **A** is

$$\mathbf{P_i} = \frac{w_i}{\sum_{i=1}^{3} w_i}$$

$$\mathbf{P_i} = \begin{pmatrix} 0.67 \\ 0.27 \\ 0.06 \end{pmatrix}$$