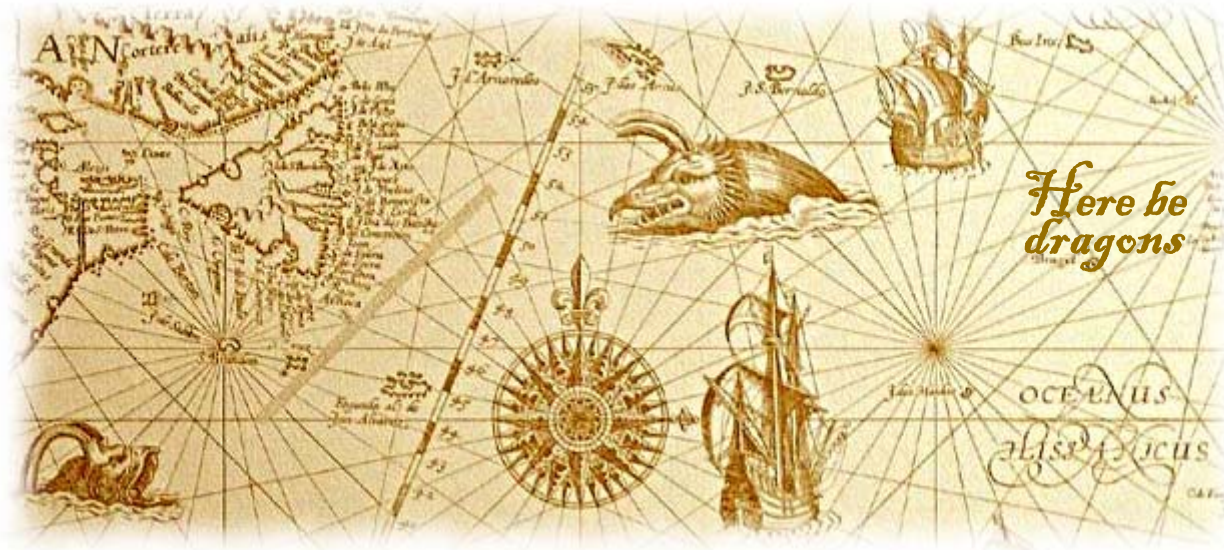


# Confusion between AND & OR

## Beware inappropriate methods for combining risk factors

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# OVERVIEW

Context: risk maps for biological invasions

Component risks

Risk and probability

Difference between AND and OR

Pareto dominance methods

Example: Mediterranean fruit fly

Conclusions

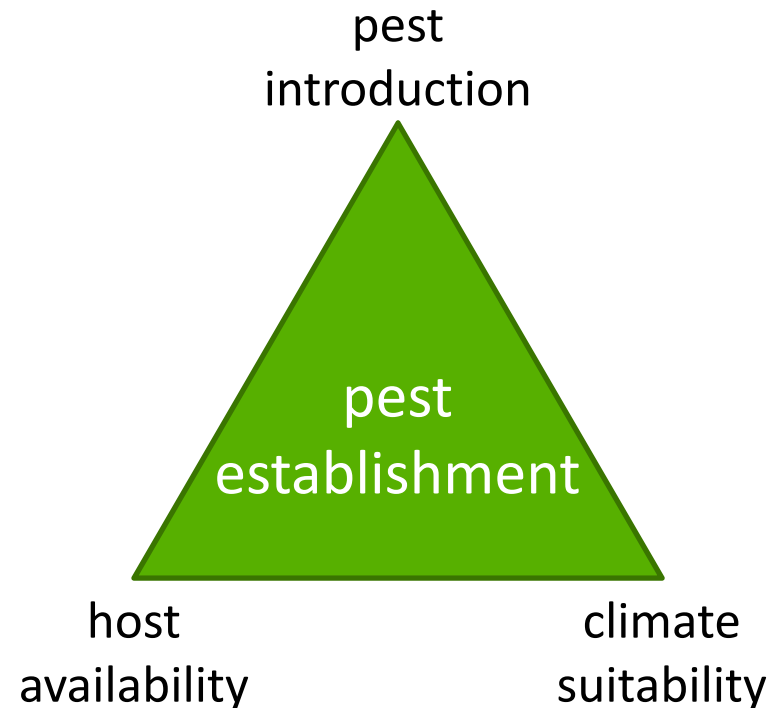


## CONTEXT: BIOLOGICAL INVASIONS

Invasive species damage  
>US\$1.4 trillion /year  
≈ 5% of global GDP\*

Invasion requires

- introduction pathway
- host plants available
- suitable climate



**Invasion triangle**

\*Pimentel *et al.* (2001)  
Agriculture, Ecosystems & Environment 84: 1–20.

# DIFFERENT WAYS OF COMBINING RISK MAPS

## Add values

- Krist et al. (2010)
- early NAPPFAST\* risk maps



## Multiply values

- Dupin et al. (2011)
- CLIMEX algorithms
- later NAPPFAST risk maps



## Pareto dominance

- Magarey et al. (2011)
- recent NAPPFAST risk maps



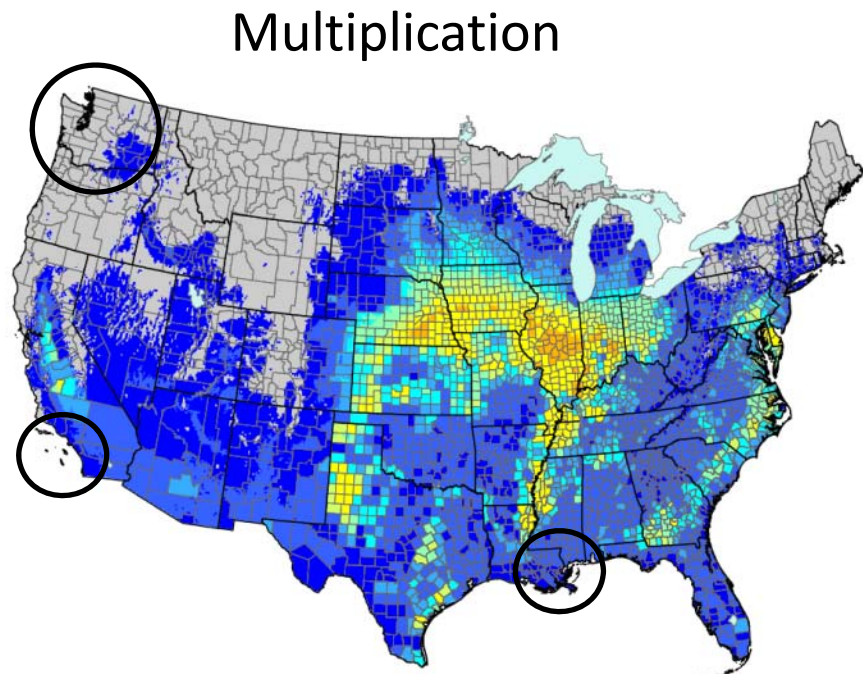
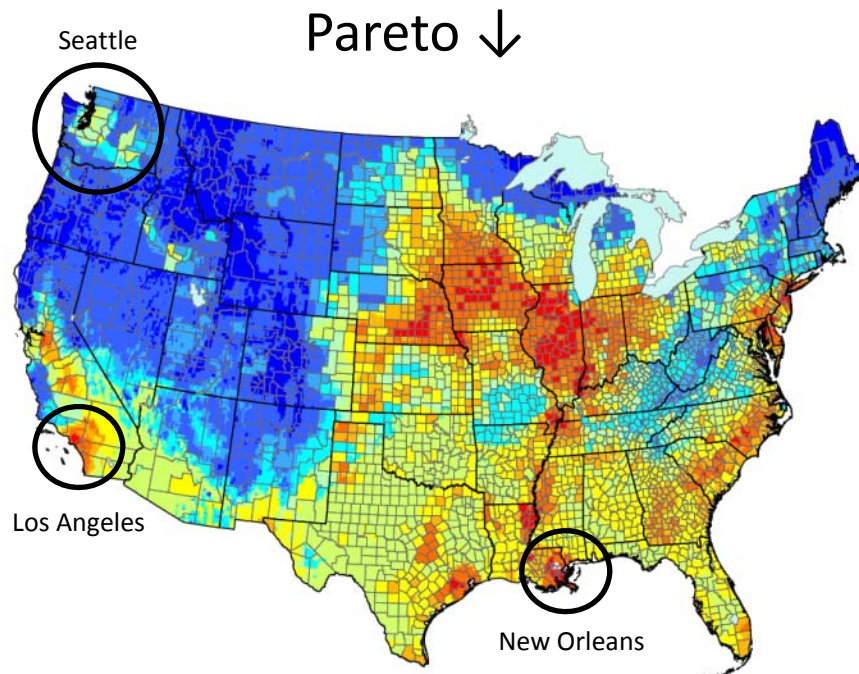
\*<http://nappfast.org/>



# COMBINING INDEPENDENT PROBABILITIES

e.g. Late wilt of corn, *Harpophora maydis*

introduction potential, climate suitability, host availability



# RISK AND PROBABILITY

Risk is a vague concept for many managers (and researchers!)

Here, risk = the **relative probability** of a **specific event** occurring

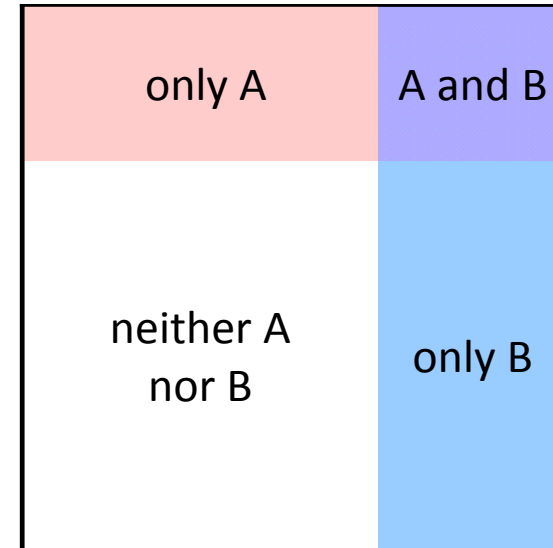
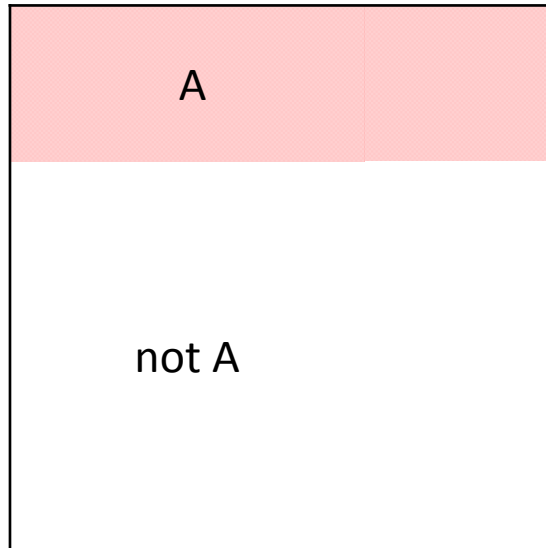
may not know true value  
(small, large uncertainty)

often poorly defined  
and/or understood

but can compare different events  
or locations = “risk maps”



# COMBINING INDEPENDENT PROBABILITIES



# COMBINING INDEPENDENT PROBABILITIES

Independent events A and B

Probability of event A =  $P_A$

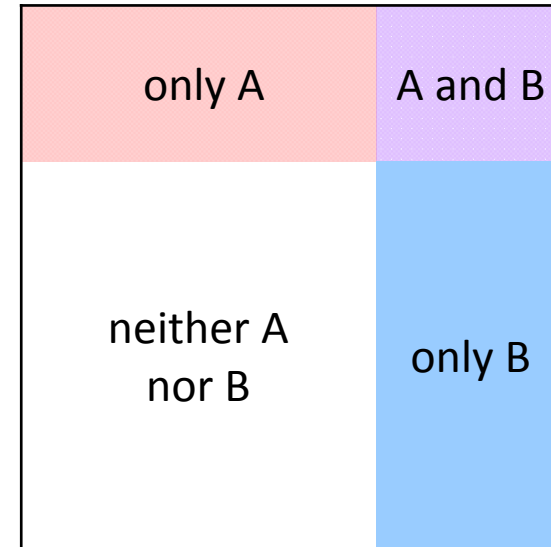
Probability of event B =  $P_B$

$$P_{A \text{ and } B} = P_A \times P_B$$

$$\begin{aligned} P_{A \text{ or } B} &= P_A + P_B - P_{A \text{ and } B} \\ &= P_A + P_B - P_A \times P_B \end{aligned}$$

If  $P_A$  and  $P_B$  are both small then  $P_{A \text{ and } B}$  is negligible, so

$$P_{A \text{ or } B} \approx P_A + P_B$$





# COMBINING INDEPENDENT RISKS: AND

Independent events A and B

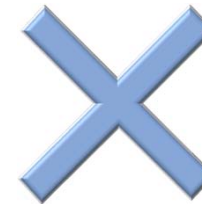
$$\text{Risk of event A: } R_A = w_A \times P_A \quad P_A = R_A / w_A$$

$$\text{Risk of event B: } R_B = w_B \times P_B \quad P_B = R_B / w_B$$

$$\begin{aligned} P_{A \text{ and } B} &= P_A \times P_B \\ &= R_A / w_A \times R_B / w_B \end{aligned}$$

$$w_A \times w_B \times P_{A \text{ and } B} = R_A \times R_B = R_{A \text{ and } B}$$

Relative risk of A and B is relative risk of A  $\times$  relative risk of B  
and weightings don't matter



**AND: multiply relative risks**

## COMBINING INDEPENDENT RISKS: OR

$$P_{A \text{ or } B} \approx P_A + P_B \quad \text{if } P_A \times P_B \text{ negligible}$$
$$\approx R_A/w_A + R_B/w_B$$

Can only simplify if  $w_A = w_B = w$

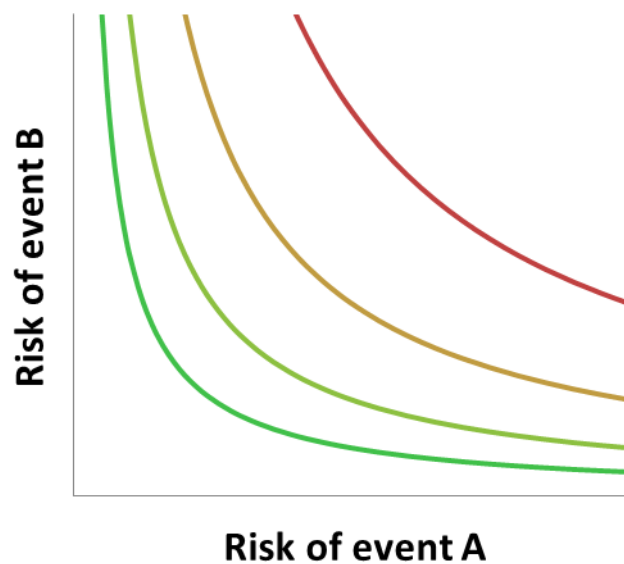
$$w \times P_{A \text{ or } B} \approx R_A + R_B = R_{A \text{ or } B}$$



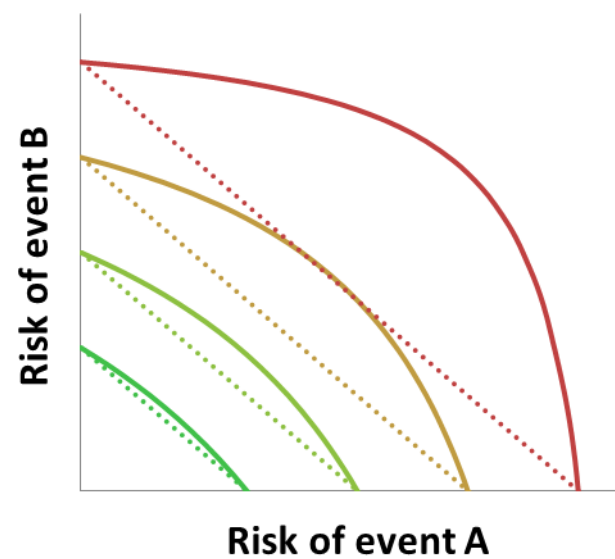
Relative risk of A or B is relative risk of A + relative risk of B  
if the underlying probabilities are small and weightings equal

**OR: add relative risks**  
**but only if underlying probabilities are small**  
**and on the same scale**

## AND vs OR



**AND**



**OR**

# COMPOSITE RISKS

Invasion risk for a single species:

$$R_{\text{introduction and establishment}} = R_{\text{introduction}} \times R_{\text{establishment}}$$



Invasion risk for several species:

$$R_{\text{invasion by species 1 or 2}} \approx R_{\text{invasion by species 1}} + R_{\text{invasion by species 2}}$$



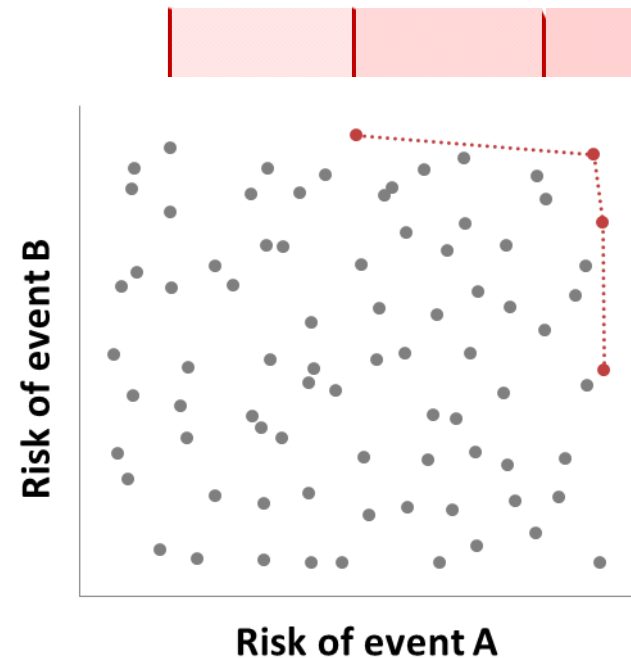
assuming that probabilities are low  
and invasion risks quantified the same way

# PARETO FRONTIER

Optimisation under uncertainty

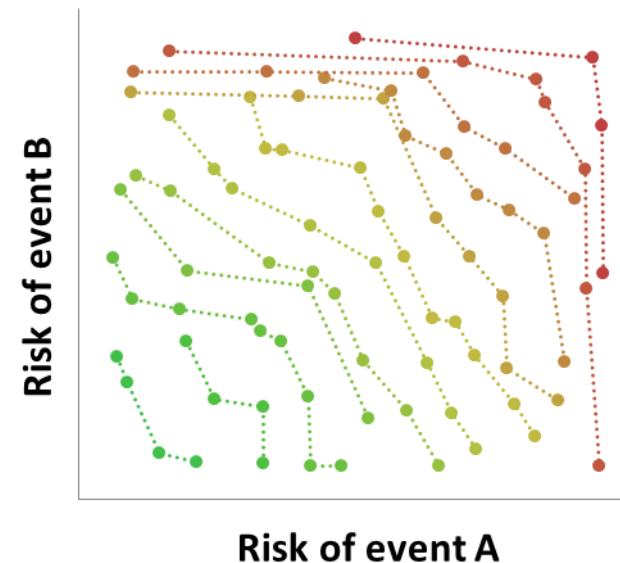
What is the minimal set of points that must contain the optimum?

What set of sites contains that with the highest (or lowest) risk, regardless of risk weightings?



## PARETO ↓

1. Find maximum Pareto frontier
2. Assign highest risk to these sites
3. Remove them from data set
4. Repeat, assigning lower risk each time

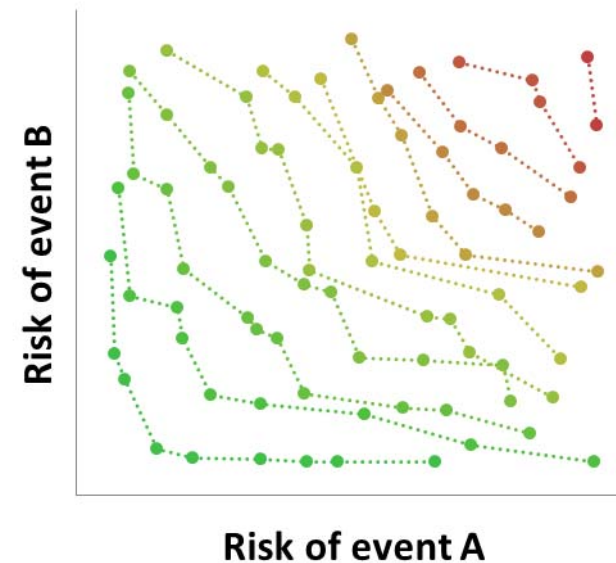


Magarey *et al.* (2011) Risk maps for targeting exotic plant pest detection programs in the United States. EPPO Bulletin 41: 46–56.



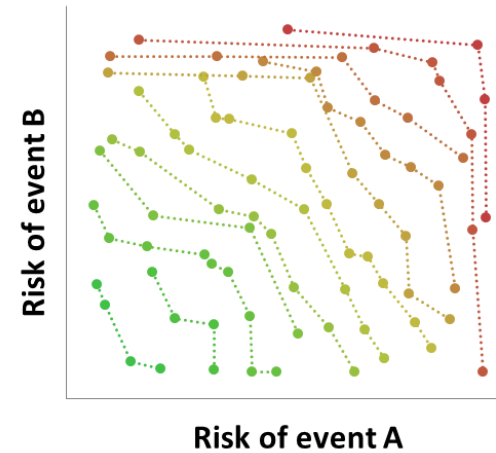
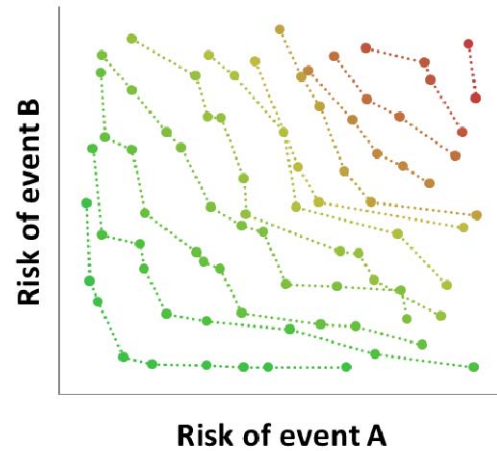
## PARETO ↑

1. Find **minimum** Pareto frontier
2. Assign **lowest** risk to these sites
3. Remove them from data set
4. Repeat, assigning **higher** risk each time



## PARETO ↑ & PARETO ↓

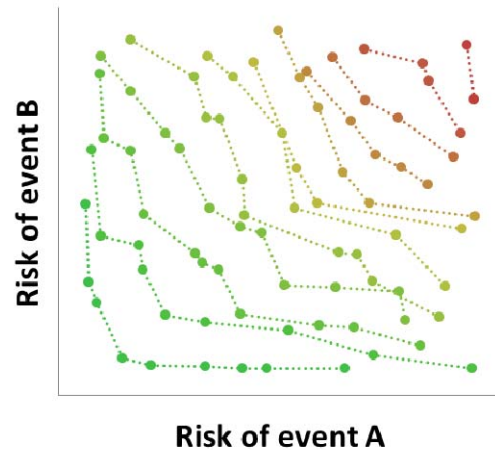
Pareto ↑



Pareto ↓

# PARETO ↑ & PARETO ↓ vs AND & OR

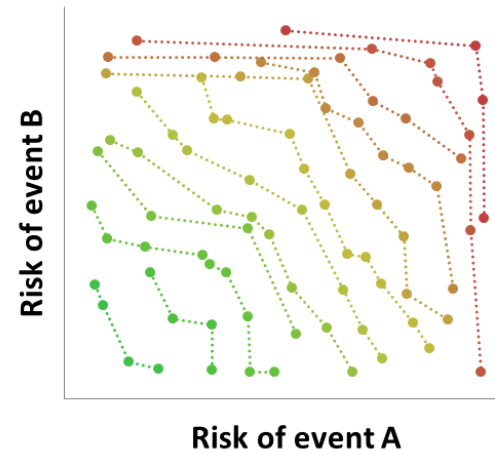
Pareto ↑



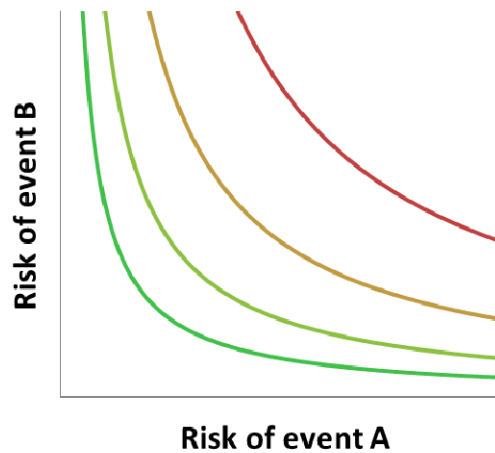
Risk of event B

Risk of event A

Pareto ↓



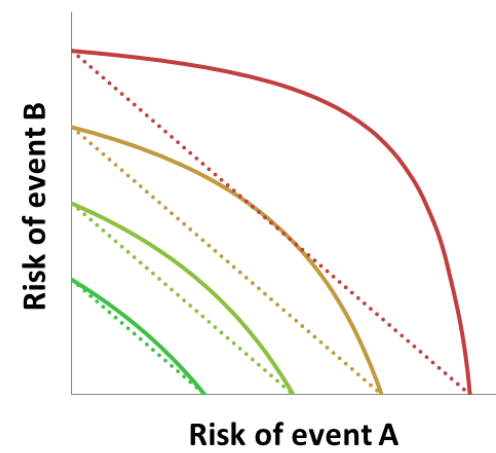
AND



Risk of event B

Risk of event A

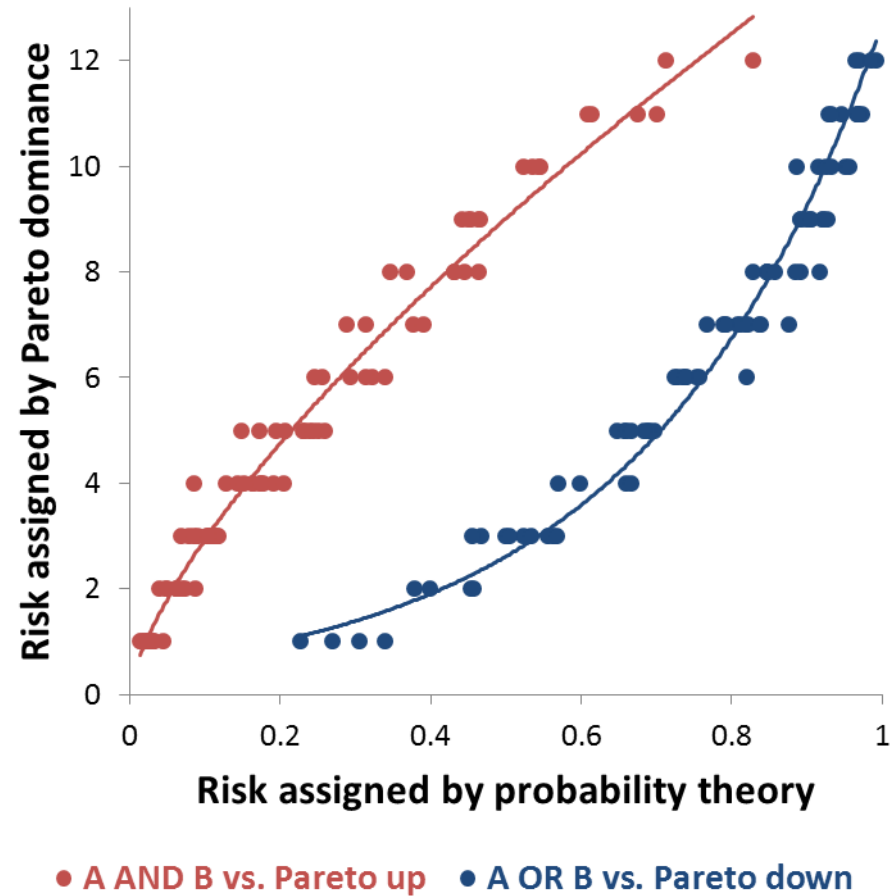
OR



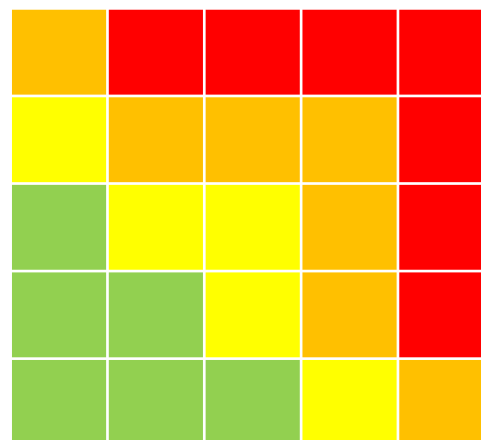
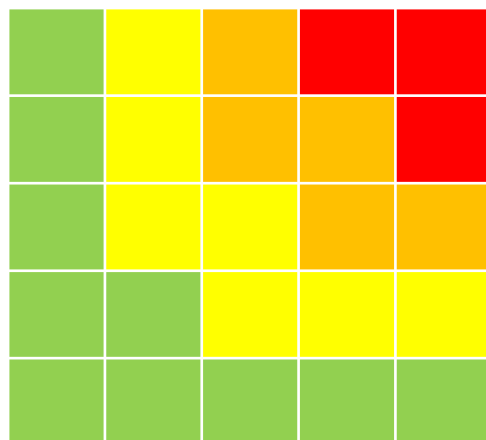
## PARETO $\uparrow$ & PARETO $\downarrow$ vs AND & OR

Pareto  $\uparrow \approx$  AND

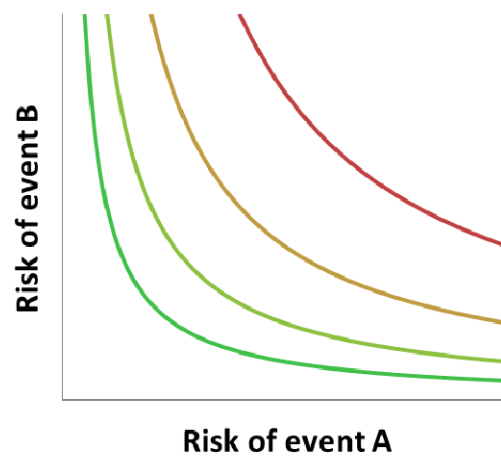
Pareto  $\downarrow \approx$  OR



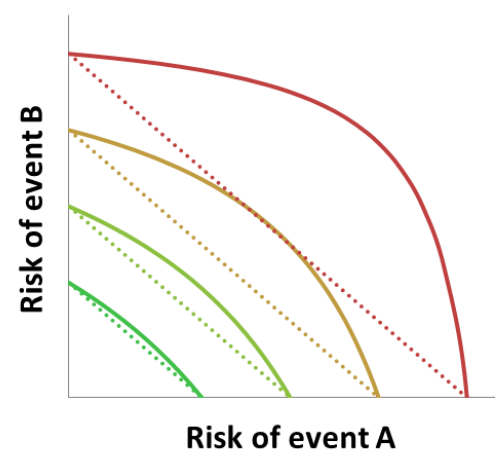
# AND & OR IN RISK MATRICES



**AND**



**OR**

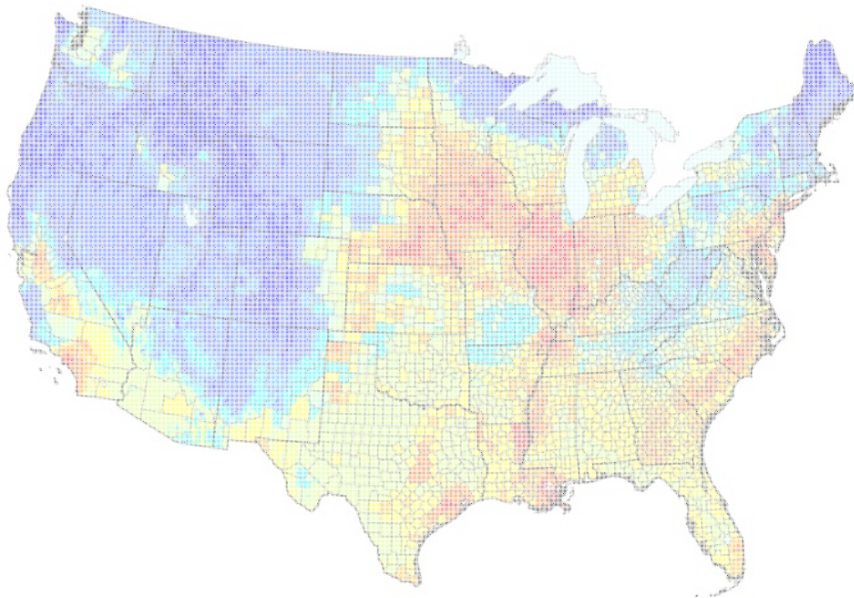


# COMBINING INDEPENDENT PROBABILITIES

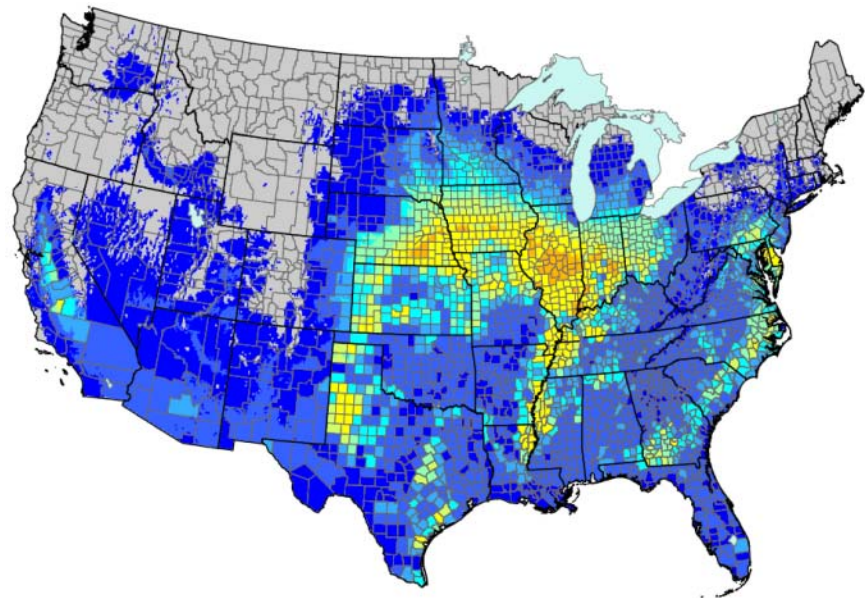
e.g. Late wilt of corn, *Harpophora maydis*

introduction potential, climate suitability, host availability

Pareto  $\downarrow \approx$  OR



Multiplication = **AND**





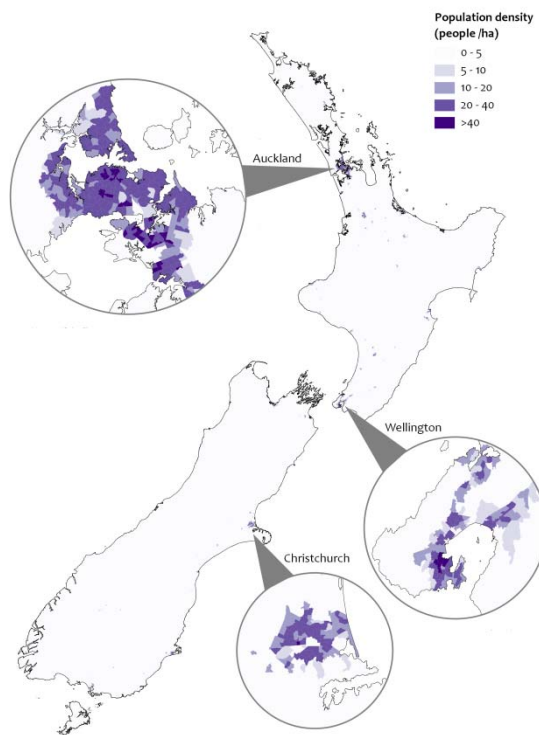
## EXAMPLE: WHERE TO LOOK FOR MEDFLY?

Mediterranean fruit fly (*Ceratitis capitata*)

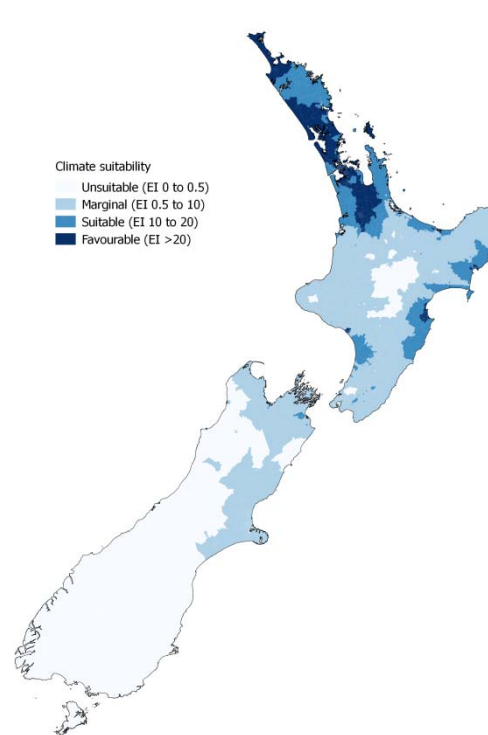


# RISK OF MEDFLY ENTRY AND ESTABLISHMENT

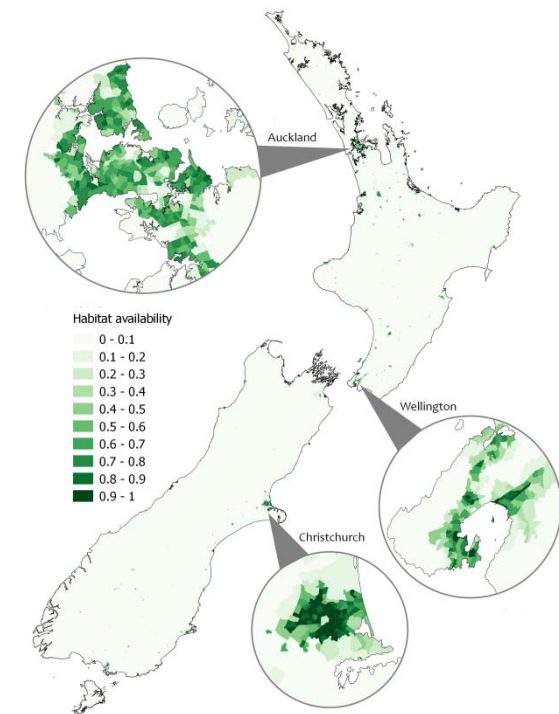
Relative probability  
of entry



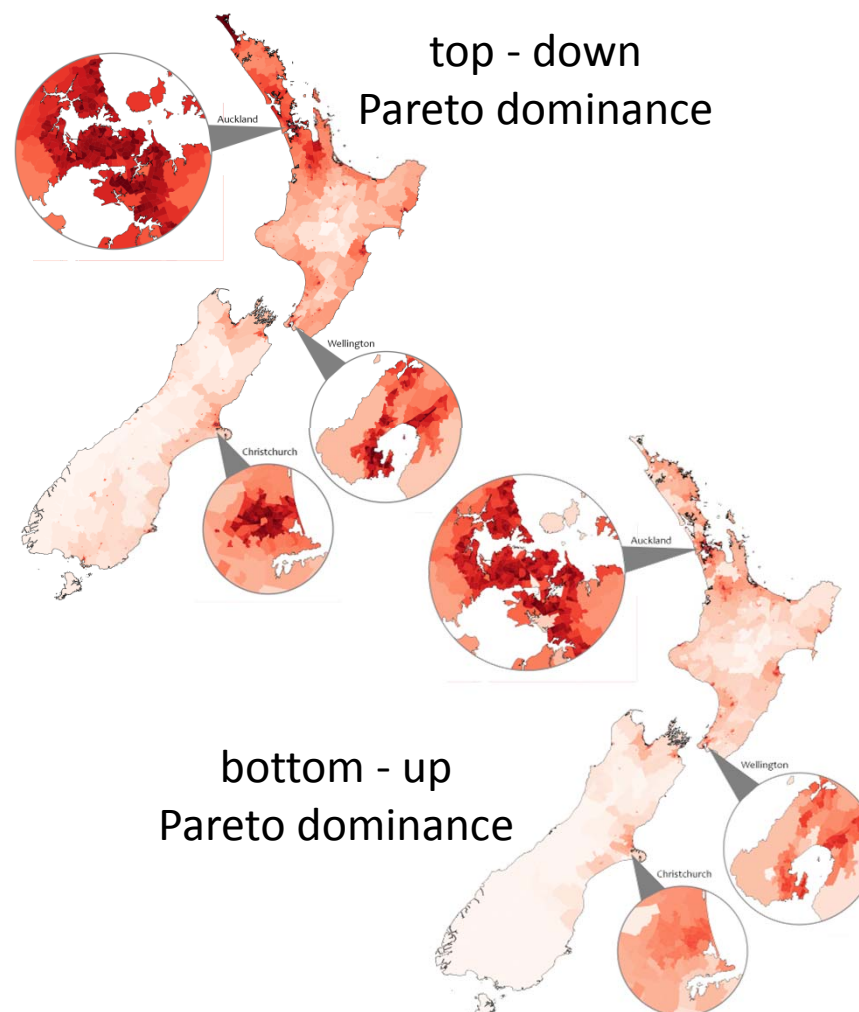
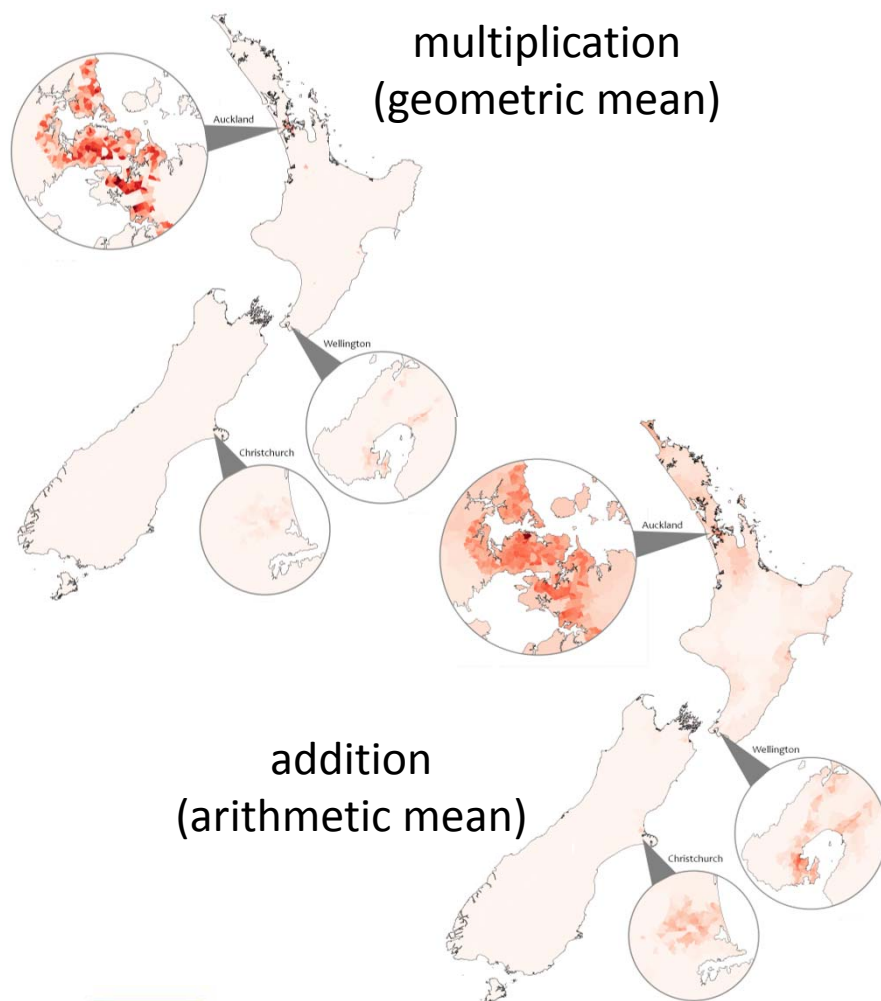
Climate suitability  
for establishment



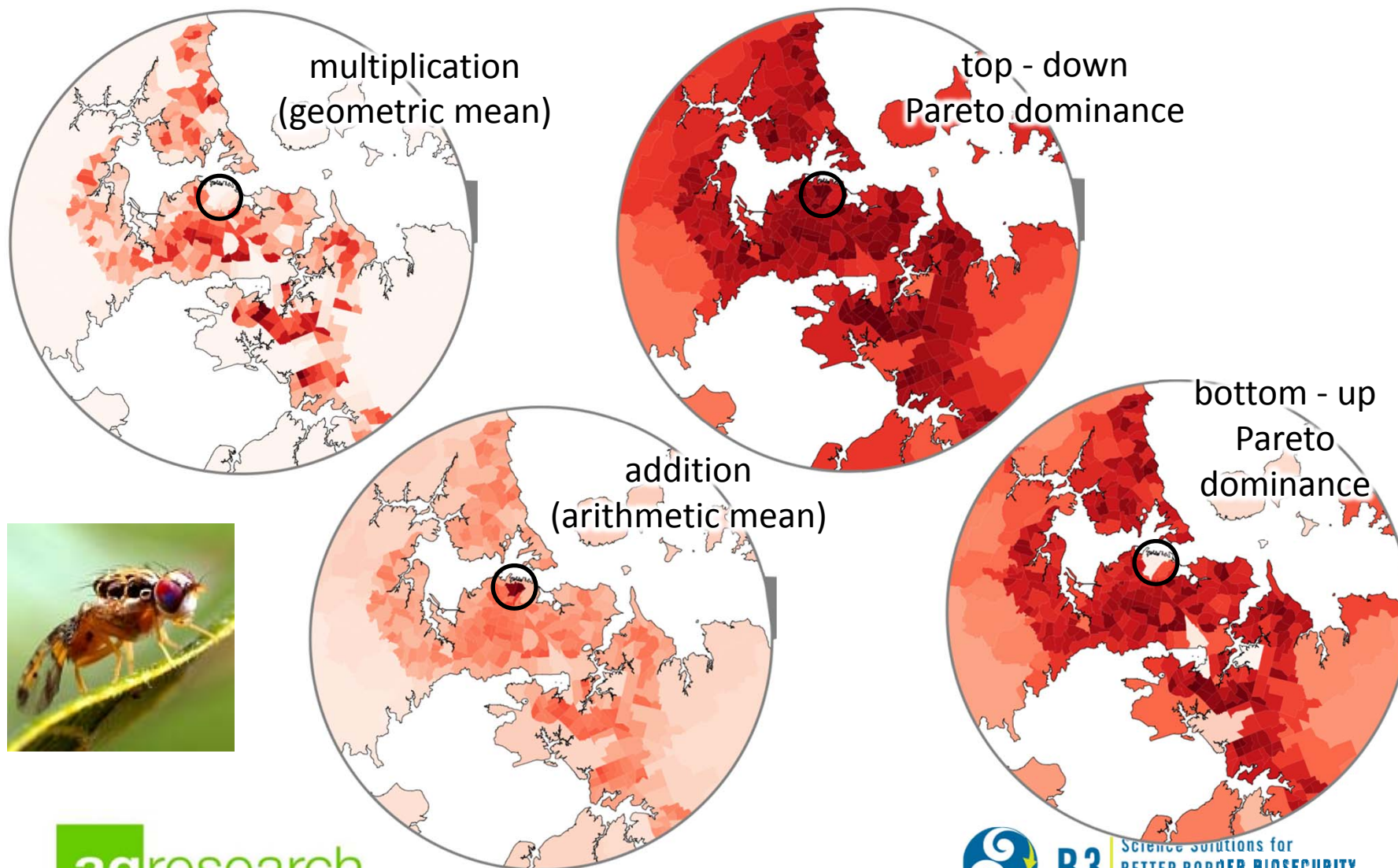
Habitat availability  
for establishment



# COMBINING COMPONENT RISK MAPS FOR MEDFLY



# COMBINING COMPONENT RISK MAPS FOR MEDFLY





# CONCLUSIONS

AND: multiply relative risks

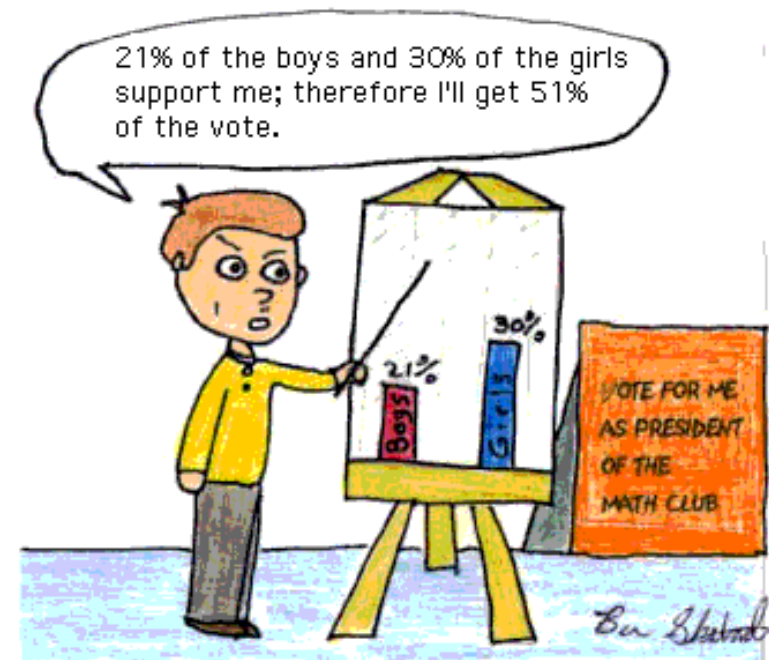
OR: add relative risks (if probabilities small and on same scale)

Analysts:

- Be clear about what the risk actually IS that you are mapping
- Combine risk maps appropriately

Decision makers:

- Understand risk maps before acting on them



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(<http://b3nz.org>)

